the moon in paſſing from A to I will continually change the plane of her motion. In what manner this change proceeds, I ſhall now particularly explain.

25. LET the plane, which touches the line AKI in the point K (in fig. 99.) interſect the plane of the earth's orbit in the line LTM. Then, becauſe the line AKI is concave to the plane ABC, it falls wholly between that plane, and the plane which touches it in K; ſo that the plane MKL will cut the plane AEC, before it meets with the plane of the earth's motion; ſuppoſe in the line YT, and the point A will fall between K and L. With a ſemidiameter equal to TY or TL deſcribe the ſemicircle LYM. Now to a ſpectator on the earth the moon, when in A, will appear to move in the circle AECF, and, when in K, will appear to be moving in the ſemicircle LYM. The earth's motion is performed in the plane of this ſcheme, and to a ſpectator on the earth the ſun will appear always moving in that plane. We may therefore refer the apparent motion of the ſun to the circle ABCD, deſcribed in this plane about the earth. But the points where this circle, in which the ſun ſeems to move, interſects the circle in which the moon is ſeen at any time to move, are called the nodes of the moon's orbit at that time. When the moon is ſeen moving in the circle AECD, the points A and C are the nodes of the orbit; when ſhe appears in the ſemicircle LYM, then L and M are the nodes. Now here it appears, from what has been ſaid, that while the moon has moved from A to K, one of the nodes has been carried from A to L, and the other as much from C to M. But the motion from A to L, and from C to

M, is backward in regard to the motion of the moon, which is the other way from A to K, and from thence toward C.

26. FARTHER the angle, which the plane, wherein the moon at any time appears, makes with the plane of the earth's motion, is called the inclination of the moon's orbit at that time. And I ſhall now proceed to ſhew, that this inclination of the orbit, when the moon is in K, is leſs than when ſhe was in A; or, that the plane LYM, which touches the line of the moon's motion in K, makes a leſs angle with the plane of the earth's motion or with the circle ABCD, than the plane AEC makes with the ſame. The ſemicircle LYM interſects the ſemicircle AEC in Y; and the arch AY is leſs than LY, and both together leſs than half a circle. But it is demonſtrated by the writers on that part of aſtronomy, which is called the doctrine of the ſphere, that when a triangle is made, as here, by three arches of circles AL, AY, and YL, the angle under YAB without the triangle is greater than the angle under YLA within, if the two arches AY, YL taken together do not amount to a ſemicircle; if the two arches make a complete ſemicircle, the two angles will be equal; but if the two arches taken together exceed a ſemicircle, the inner angle under YLA is greater than the other [a]. Here therefore the two arches AY and LY together being leſs than a ſemicircle, the angle under ALY is leſs, than the angle under BAE. But from the doctrine of the ſphere it is alſo evident, that the angle under ALY is equal to that, in which the plane of the

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[a] Menelai Sphaeric. Lib. I. prop. 10.

circle L Y K M, that is, the plane which touches the line A K G H I in K, is inclined to the plane of the earth's motion A B C; and the angle under B A E is equal to that, in which the plane A E C is inclined to the ſame plane. Therefore the inclination of the former plane is leſs than the inclination of the latter.

27. SUPPOSE now the moon to be advanced to the point G (in fig. 100.) and in this point to be diſtant from its node a quarter part of the whole circle; or in other words, to be in the midway between its two nodes. And in this caſe the nodes will have receded yet more, and the inclination of the orbit be ſtill more diminiſhed: for ſuppoſe the line A K G H I to be touched in the point G by a plane paſſing through the earth T: let the interſection of this plane with the plane of the earth's motion be the line W T O, and the line T P its interſection with the plane L K M. In this plane let the circle N G O be deſcribed with the ſemidiameter T P or N T cutting the other circle L K M in P. Now the line A K G I is convex to the plane L K M, which touches it in K; and therefore the plane N G O, which touches it in G, will interſect the other touching plane between G and K; that is, the point P will fall between thoſe two points, and the plane continued to the plane of the earth's motion will paſs beyond L; ſo that the points N and O, or the places of the nodes, when the moon is in G, will be farther from A and C than L and M, that is, will have moved farther backward. Beſides, the inclination of the plane N G O to the plane of the earth's motion A B C is leſs, than the inclination of the plane L K M to the ſame; for here alſo the two arches L P and N P taken together are leſs

than a ſemicircle, each of theſe arches being leſs than a quarter of a circle; as appears, becauſe GN, the diſtance of the moon in G from its node N, is here ſuppoſed to be a quarter part of a circle.

28. AFTER the moon is paſſed beyond G, the caſe is altered; for then theſe arches will be greater than quarters of the circle, by which means the inclination will be again increaſed, tho' the nodes ſtill go on to move the ſame way. Suppoſe the moon in H, (in fig. 101.) and that the plane, which touches the line AKGI in H, interſects the plane of the earth's motion in the line QTR, and the plane NGO in the line TV, and beſides that the circle QHR be deſcribed in that plane; then, for the ſame reaſon as before, the point V will fall between H and G, and the plane RVQ will paſs beyond the laſt plane OVN, cauſing the points Q and R to fall farther from A and C than N and O. But the arches NV, VQ are each greater than a quarter of a circle, NV the leaſt of them being greater than GN, which is a quarter of a circle; and therefore the two arches NV and VQ together exceed a ſemicircle; conſequently the angle under BQV will be greater, than that under BNV.

29. IN the laſt place, when the moon is by this attraction of the ſun, drawn at length into the plane of the earth's motion, the node will have receded yet more, and the inclination be ſo much increaſed, as to become ſomewhat more than at firſt: for the line AKGHI being convex to all the planes, which touch it, the part HI will wholly fall between

33. Let AGCH (in fig. 103.) be a circle deſcribed in the plane of the earth's motion, having the earth in T for its center. Let the point oppoſite to the ſun be A, and the point G a fourth part of the circle diſtant from A. Let the nodes of the moon's orbit be ſituated in the line BTD, and B the node, falling between A, the place where the moon would be in the full, and G the place where the moon would be in the quarter. Suppoſe BEDF to be the plane, in which the moon eſſays to move, when it proceeds from the point B. Becauſe the moon in B is more diſtant from the ſun than the earth, it ſhall be leſs attracted by the ſun, and ſhall not deſcend towards the ſun ſo faſt as the earth: conſequently it ſhall quit the plane BEDF, which we ſuppoſe to accompany the earth, and deſcribe the line BIK convex thereto, till ſuch time as it comes to the point K, where it will be in the quarter: but from thenceforth being more attracted than the earth, the moon ſhall change its courſe, and the following part of the path it deſcribes ſhall be concave to the plane BED or BGD, and ſhall continue concave to the plane BGD, till it croſſes that plane in L, juſt as in the preceding caſe. Now I ſay, while the moon is paſſing from B to K, the nodes, contrary to what was found in the foregoing caſe, will proceed forward, or move the ſame way with the moon [a]; and at the ſame time the inclination of the orbit will increaſe [b].

34. When the moon is in the point I, let the plane MIN paſs through the earth T, and touch the path of the

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[a] Vid. Newt. Princ. Lib. III. prop 30. p. 440.

[b] Ibid. Lib. I. prop. 66. coroll. [illegible]

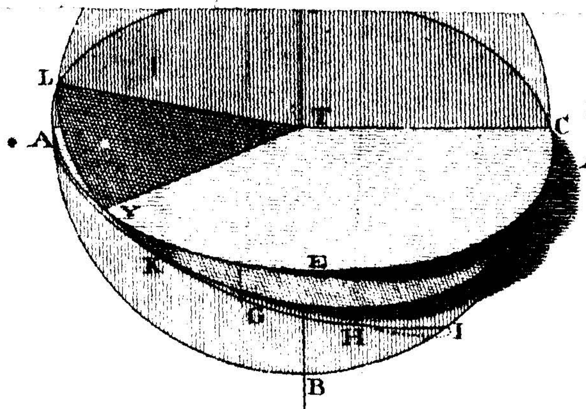

Fig. 99.

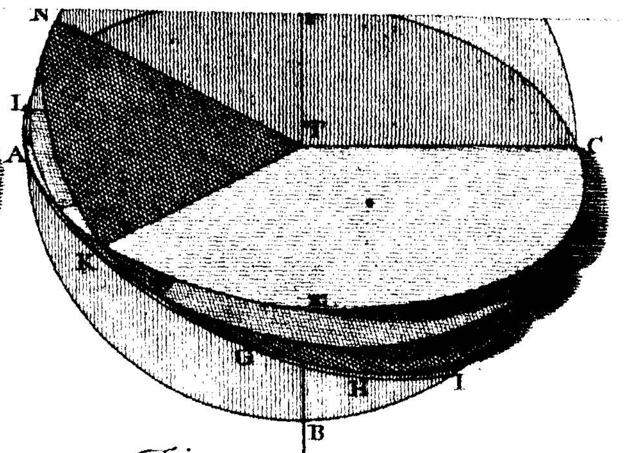

Fig. 100.

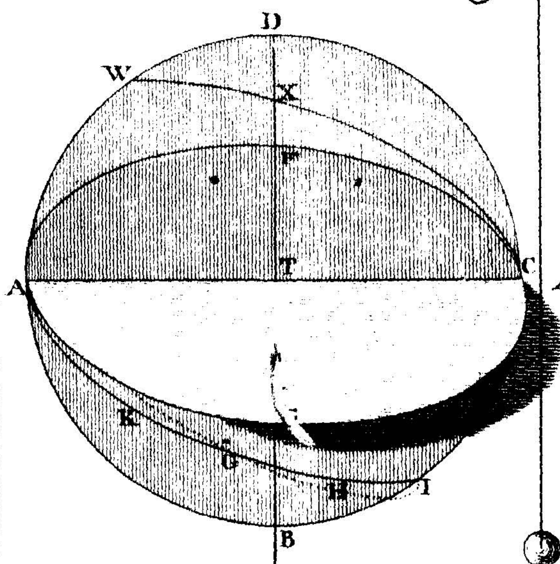

Fig. 98.

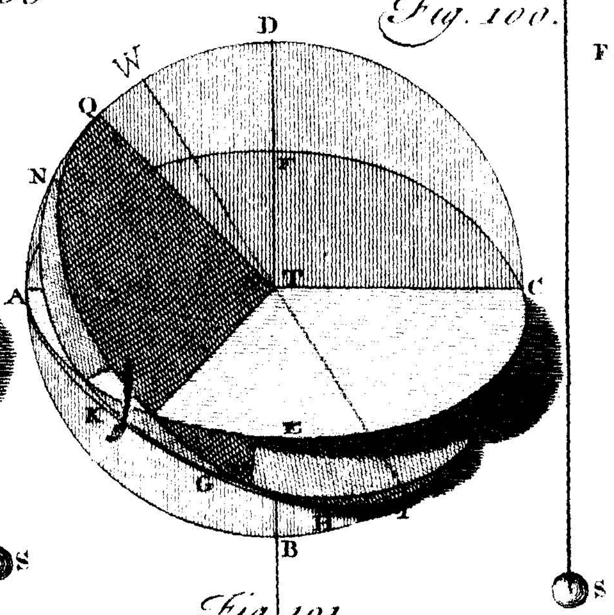

Fig. 101.

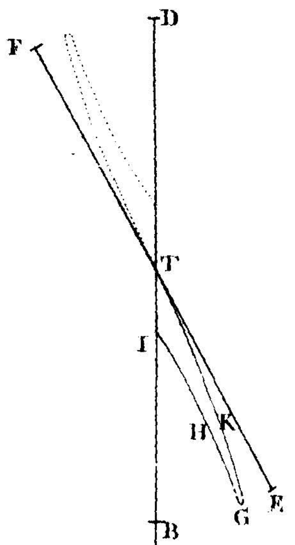

Fig. 102.

moon in I, cutting the plane of the earth's motion in the line MTN, and the plane BED in the line TO. Becauſe the line BIK is convex to the plane BED, which touches it in B, the plane NIM muſt croſs the plane DEB, before it meets the plane CGB; and therefore the point M will fall from B towards G, and the node of the moon's orbit being tranſlated from B to M is moved forward.

35. I say farther, the angle under OMG, which the plane MON makes with the plane BGC, is greater than the angle under OBG, which the plane BOD makes with the ſame. This appears from what has been already explained; becauſe the arches BO, OM are each leſs than the quarter of a circle, and therefore taken both together are leſs than a ſemicircle.

36. Again, when the moon is come to the point K in its quarter, the nodes will be advanced yet farther forward, and the inclination of the orbit alſo more augmented. Hitherto the moon's motion has been referred to the plane, which paſſing through the earth touches the path of the moon in the point, where the moon is, according to what was aſſerted at the beginning of this diſcourſe upon the nodes, that it is the cuſtom of aſtronomers ſo to do. But here in the point K no ſuch plane can be found; on the contrary, ſeeing the line of the moon's motion on one ſide the point K is convex to the plane BED, and on the other ſide concave to the ſame, no plane can paſs through the points T and K, but will cut the line BKL in that point. Therefore inſtead

of ſuch a touching plane, we muſt here make uſe of what is equivalent, the plane PKQ, with which the line BKL ſhall make a leſs angle than with any other plane; for this plane does as it were touch the line BK in the point K, ſince it ſo cuts it, that no other plane can be drawn ſo, as to paſs between the line BK and the plane PKQ. But now it is evident, that the point P, or the node, is removed from M towards G, that is, has moved yet farther forward; and it is likewiſe as manifeſt, that the angle under KPG, or the inclination of the moon's orbit in the point K, is greater than the angle under IMG, for the reaſon ſo often aſſigned.

37. After the moon has paſſed the quarter, the path of the moon being concave to the plane AGCH, the nodes, as in the preceding caſe, ſhall recede, till the moon arrives at the point L; which ſhews, that conſidering the whole time of the moon's paſſing from B to L, at the end of that time the nodes ſhall be found to have receded, or to be placed backwarder, when the moon is in L, than when it was in B. For the moon takes a longer time in paſſing from K to L, than in paſſing from B to K; and therefore the nodes continue to recede a longer time, than they moved forwards; ſo that their receſs muſt ſurmount their advance.

38. In the ſame manner, while the moon is in its paſſage from K to L, the inclination of the orbit ſhall diminiſh, till the moon comes to the point, in which it is one quarter part of a circle diſtant from its node; ſuppoſe in the point R; and from that time the inclination ſhall again increaſe.

Since therefore the inclination of the orbit increaſes, while the moon is paſſing from B to K, and diminiſhes itſelf again only, while the moon is paſſing from K to R, and then augments again, till the moon arrive in L; while the moon is paſſing from B to L, the inclination of the orbit is much more increaſed than diminiſhed, and will be diſtinguiſhably greater, when the moon is come to L, than when it ſet out from B.

39. IN like manner, while the moon is paſſing from L on the other ſide the plane A G C H, the node ſhall advance forward, as long as the moon is between the point L and the next quarter; but afterwards it ſhall recede, till the moon come to paſs the plane A G C H again in the point V, between B and A: and becauſe the time between the moon's paſſing from L to the next quarter is leſs, than the time between that quarter and the moon's coming to the point V, the node ſhall have more receded than advanced; ſo that the point V will be nearer to A, than L is to C. So alſo the inclination of the orbit, when the moon is in V, will be greater, than when the moon was at L; for this inclination increaſes all the time the moon is between L and the next quarter; it decreaſes only while the moon is paſſing from this quarter to the mid way between the two nodes, and from thence increaſes again during the whole paſſage through the other half of the way to the next node.

40. THUS we have traced the moon from her node in the quarter, and ſhewn, that at every period of the moon the nodes will have receded, and thereby will have approached

toward a conjunction with the ſun. But this conjunction will be much forwarded by the viſible motion of the ſun itſelf. In the laſt ſcheme the ſun will appear to move from S toward W. Suppoſe it appeared to have moved from S to W, while the moon's node has receded from B to V, then drawing the line W T X, the arch V X will repreſent the diſtance of the line drawn between the nodes from the ſun, when the moon is in V; whereas the arch B A repreſented that diſtance, when the moon was in B. This viſible motion of the ſun is much greater, than that of the node; for the ſun appears to revolve quite round each year, and the node is near 19 years in making one revolution. We have alſo ſeen, that when the node was in the quadrature, the inclination of the moon's orbit decreaſed, till the moon came to the conjunction, or oppoſition, according to which node it ſet out from; but that afterwards it again increaſed, till it became at the next node rather greater than at the former. When the node is once removed from the quarter nearer to a conjunction with the ſun, the inclination of the moon's orbit, when the moon comes into the node, is more ſenſibly greater, than it was in the node preceding; the inclination of the orbit by this means more and more increaſing till the node comes into conjunction with the ſun; at which time it has been ſhewn above, that the ſun has no power to change the plane of the moon's motion; and conſequently has no effect either on the nodes, or on the inclination of the orbit.

41. As ſoon as the nodes, by the action of the ſun, are got out of conjunction toward the other quarters, they begin

again to recede as before; but the inclination of the orbit in the appulſe of the moon to each ſucceeding node is leſs than at the preceding, till the nodes come again into the quarters. This will appear as follows. Let A (in fig. 104.) repreſent one of the moon's nodes placed between the point of oppoſition B and the quarter C. Let the plane A D E paſs through the earth T, and touch the path of the moon in A. Let the line A F G H be the path of the moon in her paſſage from A to H, where ſhe croſſes again the plane of the earth's motion. This line will be convex toward the plane A D E, till the moon comes to G, where ſhe is in the quarter; and after this, between G and H, the ſame line will be concave toward this plane. All the time this line is convex toward the plane A D E, the nodes will recede; and on the contrary proceed, while it is concave to that plane. All this will eaſily be conceived from what has been before ſo largely explained. But the moon is longer in paſſing from A to G, than from G to H; therefore the nodes recede a longer time, than they proceed; conſequently upon the whole, when the moon is arrived at H, the nodes will have receded, that is, the point H will fall between B and E. The inclination of the orbit will decreaſe, till the moon is arrived to the point F, in the middle between A and H. Through the paſſage between F and G the inclination will increaſe, but decreaſe again in the remaining part of the paſſage from G to H, and conſequently at H muſt be leſs than at A. The like effects, both in reſpect to the nodes and inclination of the orbit, will take place in the following paſſage of the moon on the other ſide of the plane A B E C, from H, till it comes over that plane again in I.

42. Thus the inclination of the orbit is greateſt, when the line drawn between the moon's nodes will paſs through the ſun; and leaſt, when this line lies in the quarters, eſpecially if the moon at the ſame time be in conjunction with the ſun, or in the oppoſition. In the firſt of theſe caſes the nodes have no motion, in all others, the nodes will each month have receded: and this regreſſive motion will be greateſt, when the nodes are in the quarters; for in that caſe the nodes have no progreſſive motion during the whole month, but in all other caſes the nodes do at ſome times proceed forward, viz. whenever the moon is between either quarter, and the node which is leſs diſtant from that quarter than a fourth part of a circle.

43. It now remains only to explain the irregularities in the moon's motion, which follow from the elliptical figure of the orbit. By what has been ſaid at the beginning of this chapter it appears, that the power of the earth on the moon acts in the reciprocal duplicate proportion of the diſtance: therefore the moon, if undiſturbed by the ſun, would move round the earth in a true ellipſis, and the line drawn from the earth to the moon would paſs over equal ſpaces in equal portions of time. That this deſcription of the ſpaces is altered by the ſun, has been already declared. It has alſo been ſhewn, that the figure of the orbit is changed each month; that the moon is nearer the earth at the new and full, and more remote in the quarters, than it would be without the ſun. Now we muſt paſs by theſe monthly changes, and conſider the effect, which the ſun will have in the differ-

ent ſituations of the axis of the orbit in reſpect of that luminary.

44. THE action of the ſun varies the force, wherewith the moon is drawn toward the earth; in the quarters the force of the earth is directly increaſed by the ſun; at the new and full the ſame is diminiſhed; and in the intermediate places the influence of the earth is ſometimes aided, and ſometimes leſſened by the ſun. In theſe intermediate places between the quarters and the conjunction or oppoſition, the ſun's action is ſo oblique to the action of the earth on the moon, as to produce that alternate acceleration and retardment of the moon's motion, which I obſerved above to be ſtiled the variation. But beſides this effect, the power, by which the earth attracts the moon toward itſelf, will not be at full liberty to act with the ſame force, as if the ſun acted not at all on the moon. And this effect of the ſun's action, whereby it corroborates or weakens the action of the earth, is here only to be conſidered. And by this influence of the ſun it comes to paſs, that the power, by which the moon is impelled toward the earth, is not perfectly in the reciprocal duplicate proportion of the diſtance. Conſequently the moon will not deſcribe a perfect ellipſis. One particular, wherein the moon's orbit will differ from an ellipſis, conſiſts in the places, where the motion of the moon is perpendicular to the line drawn from itſelf to the earth. In an ellipſis, after the moon ſhould have ſet out in the direction perpendicular to this line drawn from itſelf to the earth, and at its greateſt diſtance from the earth, its motion would

again become perpendicular to this line drawn between itſelf and the earth, and the moon be at its neareſt diſtance from the earth, when it ſhould have performed half its period; after performing the other half of its period its motion would again become perpendicular to the forementioned line, and the moon return into the place whence it ſet out, and have recovered again its greateſt diſtance. But the moon in its real motion, after ſetting out as before, ſometimes makes more than half a revolution, before its motion comes again to be perpendicular to the line drawn from itſelf to the earth, and the moon is at its neareſt diſtance; and then performs more than another half of an intire revolution before its motion can a ſecond time recover its perpendicular direction to the line drawn from the moon to the earth, and the moon arrive again to its greateſt diſtance from the earth. At other times the moon will deſcend to its neareſt diſtance, before it has made half a revolution, and recover again its greateſt diſtance, before it has made an intire revolution. The place, where the moon is at its greateſt diſtance from the earth, is called the moon's apogeon, and the place of the leaſt diſtance the perigeon. This change of the place, where the moon ſucceſſively comes to its greateſt diſtance from the earth, is called the motion of the apogeon. In what manner the ſun cauſes the apogeon to move, I ſhall now endeavour to explain.

45. Our author ſhews, that if the moon were attracted toward the earth by a compoſition of two powers, one of which were reciprocally in the duplicate proportion of the diſtance from the earth, and the other reciprocally

in the triplicate proportion of the ſame diſtance; then, though the line deſcribed by the moon would not be in reality an ellipſis, yet the moon's motion might be perfectly explained by an ellipſis, whoſe axis ſhould be made to move round the earth; this motion being in conſequence, as aſtronomers expreſs themſelves, that is, the ſame way as the moon itſelf moves, if the moon be attracted by the ſum of the two powers; but the axis muſt move in antecedence, or the contrary way, if the moon be acted on by the difference of theſe powers. What is meant by duplicate proportion has been often explained; namely, that if three magnitudes, as A, B, and C, are ſo related, that the ſecond B bears the ſame proportion to the third C, as the firſt A bears to the ſecond B, then the proportion of the firſt A to the third C, is the duplicate of the proportion of the firſt A to the ſecond B. Now if a fourth magnitude, as D, be aſſumed, to which C ſhall bear the ſame proportion as A bears to B, and B to C, then the proportion of A to D is the triplicate of the proportion of A to B.

40. THE way of repreſenting the moon's motion in this caſe is thus. T denoting the earth (in fig. 105, 106.) ſuppoſe the moon in the point A, its apogeon, or greateſt diſtance from the earth, moving in the direction AF perpendicular to AB, and acted upon from the earth by two ſuch forces as have been named. By that power alone, which is reciprocally in the duplicate proportion of the diſtance, if the moon ſet out from the point A with a proper degree of velocity, the ellipſis AMB may be deſcribed.

ſcribed. But if the moon be acted upon by the ſum of the forementioned powers, and the velocity of the moon in the point A be augmented in a certain proportion [a]; or if that velocity be diminiſhed in a certain proportion, and the moon be acted upon by the difference of thoſe powers; in both theſe caſes the line AE, which ſhall be deſcribed by the moon, is thus to be determined. Let the point M be that, into which the moon would have arrived in any given ſpace of time, had it moved in the ellipſis AMB. Draw MT, and likewiſe CTD in ſuch ſort, that the angle under ATM ſhall bear the ſame proportion to the angle under ATC, as the velocity, with which the ellipſis AMB muſt have been deſcribed, bears to the difference between this velocity, and the velocity, with which the moon muſt ſet out from the point A in order to deſcribe the path AE. Let the angle ATC be taken toward the moon (as in fig. 105.) if the moon be attracted by the ſum of the powers; but the contrary way (as in fig. 106.) if by their difference. Then let the line AB be moved into the poſition CD, and the ellipſis AMB into the ſituation CND, ſo that the point M be tranſlated to L: then the point L ſhall fall upon the path of the moon AE.

47. The angular motion of the line AT, wereby it is removed into the ſituation CT, repreſents the motion of the apogeon; by the means of which the motion of the moon might be fully explicated by the ellipſis AMB, if the action of the ſun upon it was directed to the center of the earth, and

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[a] What this proportion is, may be known from Coroll. 2. prop 44. Lib. I. Princ. philoſ. Newton.

reciprocally in the triplicate proportion of the moon's diſtance from it. But that not being ſo, the apogeon will not move in the regular manner now deſcribed. However, it is to be obſerved here, that in the firſt of the two preceding caſes, where the apogeon moves forward, the whole centripetal power increaſes faſter, with the decreaſe of diſtance, than if the intire power were reciprocally in the duplicate proportion of the diſtance; becauſe one part only is in that proportion, and the other part, which is added to this to make up the whole power, increaſes faſter with the decreaſe of diſtance. On the other hand, when the centripetal power is the difference between theſe two, it increaſes leſs with the decreaſe of the diſtance, than if it were ſimply in the reciprocal duplicate proportion of the diſtance. Therefore if we chuſe to explain the moon's motion by an ellipſis (as is moſt convenient for aſtronomical uſes to be done, and by reaſon of the ſmall effect of the ſun's power, the doing ſo will not be attended with any ſenſible error;) we may collect in general, that when the power, by which the moon is attracted to the earth, by varying the diſtance, increaſes in a greater than in the duplicate proportion of the diſtance diminiſhed, a motion in conſequence muſt be aſcribed to the apogeon; but that when the attraction increaſes in a leſs proportion than that named, the apogeon muſt have given to it a motion in antecedence[a]. It is then obſerved by Sir Is. NEWTON, that the firſt of theſe caſes obtains, when the moon is in the conjunction and oppoſition; and the latter, when the moon is in the quarters: ſo that in the firſt the apogeon moves according to the order of the ſigns; in the other, the contrary way[a]. But, as was ſaid before, the diſturbance given to the action of the earth by the ſun in the conjunction and oppoſition being near twice as great as in the quarters[b], the apogeon will advance with a greater velocity than recede, and in the compaſs of a whole revolution of the moon will be carried in conſequence[c].

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[a] Princ. Phil. Newt. Lib. I. prop. 45. Coroll. 1.

48. It is ſhewn in the next place by our author, that when the line A B coincides with that, which joins the earth and the ſun, the progreſſive motion of the apogeon, when the moon is in the conjunction or oppoſition, exceeds the regreſſive in the quadratures more than in any other ſituation of the line A B[d]. On the contrary, when the line A B makes right angles with that, which joins the earth and ſun, the retrograde motion will be more conſiderable[e], nay is found ſo great as to exceed the progreſſive; ſo that in this caſe the apogeon in the compaſs of an intire revolution of the moon is carried in antecedence. Yet from the conſiderations in the laſt paragraph the progreſſive motion exceeds the other; ſo that in the whole the mean motion of the apogeon is in conſequence, according as aſtronomers find. Moreover, the line A B changes its ſituation with that, which joins the earth and ſun, by ſuch ſlow degrees, that the inequalities in the motion of the apogeon ariſing from this laſt conſideration, are much greater than what ariſes from the other[f]

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[a] Pr. Phil. Newt. Lib. I. prop. 66. Coroll. 7.

[b] See § 19 of this chapter.

[c] Phil. Nat. Pr. Math Lib. I. prop. 66 cor. 8.

[d] Ibid. Coroll. 8.

[e] Ibid.

[f] Ibid.

49. FARTHER, this unſteady motion in the apogeon is attended with another inequality in the motion of the moon, that it cannot be explained at all times by the ſame ellipſis. The ellipſis in general is called by aſtronomers an eccentric orbit. The point, in which the two axis's croſs, is called the center of the figure; becauſe all lines drawn through this point within the ellipſis, from ſide to ſide, are divided in the middle by this point. But the center, about which the heavenly bodies revolve, lying out of this center of the figure in one focus, theſe orbits are ſaid to be eccentric; and where the diſtance of the focus from this center bears the greateſt proportion to the whole axis, that orbit is called the moſt eccentric: and in ſuch an orbit the diſtance from the focus to the remoter extremity of the axis bears the greateſt proportion to the diſtance of the nearer extremity. Now whenever the apogeon of the moon moves in conſequence, the moon's motion muſt be referred to an orbit more eccentric, than what the moon would deſcribe, if the whole power, by which the moon was acted on in its paſſing from the apogeon, changed according to the reciprocal duplicate proportion of the diſtance from the earth, and by that means the moon did deſcribe an immoveable ellipſis; and when the apogeon moves in antecedence, the moon's motion muſt be referred to an orbit leſs eccentric. In the firſt of the two figures laſt referred to, the true place of the moon L falls without the orbit A M B, to which its motion is referred: whence the orbit A L E, truly deſcribed by the moon, is leſs incurvated in the point A, than is the orbit A M B; therefore the orbit A M B is more oblong, and differs farther from a circle, than the ellipſis would,

whoſe curvature in A were equal to that of the line ALB, that is, the proportion of the diſtance of the earth T from the center of the ellipſis to its axis will be greater in the ellipſis AMB, than in the other; but that other is the ellipſis, which the moon would deſcribe, if the power acting upon it in the point A were altered in the reciprocal duplicate proportion of the diſtance. In the ſecond figure, when the apogeon recedes, the place of the moon L falls within the orbit AMB, and therefore that orbit is leſs eccentric, than the immoveable orbit which the moon ſhould deſcribe. The truth of this is evident; for, when the apogeon moves forward, the power, by which the moon is influenced in its deſcent from the apogeon, increaſes faſter with the decreaſe of diſtance, than in the duplicate proportion of the diſtance; and conſequently the moon being drawn more forcibly toward the earth, it will deſcend nearer to it. On the other hand, when the apogeon recedes, the power acting on the moon increaſes with the decreaſe of diſtance in leſs than the duplicate proportion of the diſtance; and therefore the moon is leſs impelled toward the earth, and will not deſcend ſo low.

50. Now ſuppoſe in the firſt of theſe figures, that the apogeon A is in the ſituation, where it is approaching toward the conjunction or oppoſition of the ſun. In this caſe the progreſſive motion of the apogeon is more and more accelerated. Here ſuppoſe that the moon, after having deſcended from A through the orbit AE as far as F, where it is come to its neareſt diſtance from the earth, aſcends again up the line FG. Becauſe the motion of the apogeon is here continually more and

more accelerating, the cauſe of its motion is conſtantly upon the increaſe; that is, the power, whereby the moon is drawn to the earth, will decreaſe with the increaſe of diſtance, in the moon's aſcent from F, in a greater proportion than that wherewith it increaſed with the decreaſe of diſtance in the moon's deſcent to F. Conſequently the moon will aſcend higher than to the diſtance A T, from whence it deſcended; therefore the proportion of the greateſt diſtance of the moon to the leaſt is increaſed. And when the moon deſcends again, the power will yet more increaſe with the decreaſe of diſtance, than in the laſt aſcent it decreaſed with the augmentation of diſtance; the moon therefore muſt deſcend nearer to the earth than it did before, and the proportion of the greateſt diſtance to the leaſt yet be more increaſed. Thus as long as the apogeon is advancing toward the conjunction or oppoſition, the proportion of the greateſt diſtance of the moon from the earth to the leaſt will continually increaſe; and the elliptical orbit, to which the moon's motion is referred, will be rendered more and more eccentric.

51. As ſoon as the apogeon is paſſed the conjunction with the ſun or the oppoſition, the progreſſive motion thereof abates, and with it the proportion of the greateſt diſtance of the moon from the earth to the leaſt diſtance will alſo diminiſh; and when the apogeon becomes regreſſive, the diminution of this proportion will be ſtill farther continued on, till the apogeon comes into the quarter; from thence this proportion, and the eccentricity of the orbit will increaſe again. Thus the orbit of the moon is moſt eccentric, when the apo-

geon is in conjunction with the ſun, or in oppoſition to it, and leaſt of all when the apogeon is in the quarters.

52. THESE changes in the nodes, in the inclination of the orbit to the plane of the earth's motion, in the apogeon, and in the eccentricity, are varied like the other inequalities in the motion of the moon, by the different diſtance of the earth from the ſun; being greateſt, when their cauſe is greateſt, that is, when the earth is neareſt to the ſun.

53. I ſaid at the beginning of this chapter, that Sir ISAAC NEWTON has computed the very quantity of many of the moon's inequalities. That acceleration of the moon's motion, which is called the variation, when greateſt, removes the moon out of the place, in which it would otherwiſe be found, ſomething more than half a degree [a]. In the phraſe of aſtronomers, a degree is $\frac{1}{360}$ part of the whole circuit of the moon or any planet. If the moon, without diſturbance from the ſun, would have deſcribed a circle concentrical to the earth, the ſun will cauſe the moon to approach nearer to the earth in the conjunction and oppoſition, than in the quarters, nearly in the proportion of 69 to 70 [b]. We had occaſion to mention above, that the nodes perform their period in almoſt 19 years. This the aſtronomers found by obſervation; and our author's computations aſſign to them the ſame period [c]. The inclination of the moon's orbit when leaſt, is an angle about $\frac{1}{18}$ part of that angle, which conſtitutes

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[a] Newt. Princ. Lib. III. prop. 29.

[b] Ibid. prop. 28.

[c] Ibid. prop. 32.

a perpendicular; and the difference between the greateſt and leaſt inclination of the orbit is determined by our author's computation to be about $\frac{1}{18}$ of the leaſt inclination [b]. And this alſo is agreeable to the obſervations of aſtronomers. The motion of the apogeon, and the changes in the eccentricity, Sir ISAAC NEWTON has not computed. The apogeon performs its revolution in about eight years and ten months. When the moon's orbit is moſt eccentric, the greateſt diſtance of the moon from the earth bears to the leaſt diſtance nearly the proportion of 8 to 7; when the orbit is leaſt eccentric, this proportion is hardly ſo great as that of 12 to 11.

54. Sir ISAAC NEWTON ſhews farther, how, by comparing the periods of the motion of the ſatellites, which revolve round Jupiter and Saturn, with the period of our moon round the earth, and the periods of thoſe planets round the ſun with the period of our earth's motion, the inequalities in the motion of thoſe ſatellites may be derived from the inequalities in the moon's motion; excepting only in regard to that motion of the axis of the orbit, which in the moon makes the motion of the apogeon; for the orbits of thoſe ſatellites, as far as can be diſcerned by us at this diſtance, appearing little or nothing eccentric, this motion, as deduced from the moon, muſt be diminiſhed.

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[b] Newt. Princ. pag. 459.

CHAP. IV.

Of COMETS.

IN the former of the two preceding chapters the powers have been explained, which keep in motion thoſe celeſtial bodies, whoſe courſes had been well determined by the aſtronomers. In the laſt chapter we have ſhewn, how thoſe powers have been applied by our author to the making a more perfect diſcovery of the motion of thoſe bodies, the courſes of which were but imperfectly underſtood; for ſome of the inequalities, which we have been deſcribing in the moon's motion, were unknown to the aſtronomers. In this chapter we are to treat of a third ſpecies of the heavenly bodies, the true motion of which was not at all apprehended before our author writ; in ſo much, that here Sir ISAAC NEWTON has not only explained the cauſes of the motion of theſe bodies, but has performed alſo the part of an aſtronomer, by diſcovering what their motions are.

2. THAT theſe bodies are not meteors in our air, is manifeſt; becauſe they riſe and ſet in the ſame manner, as the ſun and ſtars. The aſtronomers had gone ſo far in their inquiries concerning them, as to prove by their obſervations, that they moved in the etherial ſpaces far beyond the moon; but they had no true notion at all of the path, which they deſcribed. The moſt prevailing opinion before

our author was, that they moved in ſtraight lines; but in what part of the heavens was not determined. DES CARTES [a] removed them far beyond the ſphere of Saturn, as finding the ſtraight motion attributed to them, inconſiſtent with the vortical fluid, by which he explains the motions of the planets, as we have above related [b]. But Sir ISAAC NEWTON diſtinctly proves from aſtronomical obſervation, that the comets paſs through the region of the planets, and are moſtly inviſible at a leſs diſtance, than that of Jupiter [c].

3. AND from hence finding the comets to be evidently within the ſphere of the ſun's action, he concludes they muſt neceſſarily move about the ſun, as the planets do [d]. The planets move in ellipſis's; but it is not neceſſary that every body which is influenced by the ſun, ſhould move in that particular kind of line. However our author proves, that the power of the ſun being reciprocally in the duplicate proportion of the diſtance, every body acted on by the ſun muſt either fall directly down, or move in ſome conic ſection; of which lines I have above obſerved, that there are three ſpecies, the ellipſis, parabola, and hyperbola [e]. If a body, which deſcends toward the ſun as low as the orbit of any planet, move with a ſwifter motion than the planet does, that body will deſcribe an orbit of a more oblong figure, than that of the planet, and have a longer axis at leaſt. The velocity of the body may be ſo great, that it

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[a] In Princ. philoſ. part. 3. § 41.

[b] Chap. 1. § 11.

[c] Newton. Princ. philoſ. Lib. III. Lemm. 4. pag. 478.

[d] Princ. philoſ. Lib. III. prop. 40.

[e] Book I. chap. 2. § 82.

ſhall move in a parabola, and having once paſſed about the ſun, ſhall aſcend for ever without returning any more: but the ſun will be placed in the focus of this parabola. With a velocity ſtill greater the body will move in an hyperbola. But it is moſt probable, that the comets move in elliptical orbits, though of a very oblong, or in the phraſe of aſtronomers, of a very eccentric form, ſuch as is repreſented in fig. 107, where S is the ſun, C the comet, and ABDE its orbit, wherein the diſtance of S and D far exceeds that of S and A. Whence it is, that they ſometimes are found at a moderate diſtance from the ſun, and appear within the planetary regions; at other times they aſcend to vaſt diſtances, far beyond the very orbit of Saturn, and ſo become inviſible. That the comets do move in this manner is proved by our author, from computations built upon the obſervations, which aſtronomers had made on many comets. Theſe computations were performed by Sir ISAAC NEWTON himſelf upon the comet, which appeared toward the latter end of the year 1680, and at the beginning of the year following[a]; but the learned Dr. HALLEY proſecuted the like computations more at large in this, and alſo in many other comets[b]. Which computations are made upon propoſitions highly worthy of our author's unparallel'd genius, ſuch as could ſcarce have been diſcovered by any one not poſſeſſed of the utmoſt force of invention;

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[a] Princ. philoſ. Lib. III. pag. 499, 500.

[b] Ibid. pag. 500, and 510, &c.

4. THOSE computations depend upon this principle, that the eccentricity of the orbits of the comets is ſo great, that if they are really elliptical, yet they approach ſo near to parabolas in that part of them, where they come under our view, that they may be taken for ſuch without ſenſible error [a]: as in the preceding figure the parabola FAG differs in the lower part of it about A very little from the ellipſis DEAB. Upon which ground our great author teaches a method of finding by three obſervations made upon any comet the parabola, which neareſt agrees with its orbit [b]

5. NOW what confirms this whole theory beyond the leaſt room for doubt is, that the places of the comets computed in the orbits, which the method here mentioned aſſigns them, agree to the obſervations of aſtronomers with the ſame degree of exactneſs, as the computations of the primary planets places uſually do; and this in comets, whoſe motions are very extraordinary [c].

6. OUR author afterwards ſhews how to make uſe of any ſmall deviation from the parabola, that ſhall be obſerved, to determine whether the orbits of the comets are elliptical or not, and ſo to diſcover if the ſame comet returns at certain periods [d]. And upon examining the comet in 1680, by the rule laid down for this purpoſe, he finds its orbit to agree more exactly to an ellipſis than

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[a] Princ. Philoſ. Lib. III. prop. 40.

[b] Ibid. prop. 41.

[c] Ibid. pag 512.

[d] Ibid. prop. 42.

to a parabola, though the ellipſis be ſo very eccentric, that the comet cannot perform its period through it in the ſpace of 500 years [a]. Upon this Dr. HALLEY obſerved, that mention is made in hiſtory of a comet, with the like eminent tail as this, having appeared three ſeveral times before; the firſt of which appearances was at the death of JULIUS CESAR, and each appearance was at the diſtance of 575 years from the next preceding. He therefore computed the motion of this comet in ſuch an elliptic orbit, as would require this number of years for the body to revolve through it; and theſe computations agree yet more perfectly with the obſervations made on this comet, than any parabolical orbit will do [b].

7. THE comparing together different appearances of the ſame comet, is the only way to diſcover certainly the true form of the orbit: for it is impoſſible to determine with exactneſs the figure of an orbit ſo exceedingly eccentric, from ſingle obſervations taken in one part of it; and therefore Sir ISAAC NEWTON [c] propoſes to compare the orbits, upon the ſuppoſition that they are parabolical, of ſuch comets as appear at different times; for if the ſame orbit be found to be deſcribed by a comet at different times, in all probability it will be the ſame comet which deſcribes it. And here he remarks from Dr. HALLEY, that the ſame orbit very nearly agrees to two appearances of a comet about the ſpace of 75 years diſtance [d]; ſo that

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[a] Newt. Princ. philoſ. edit. 2. p. 464, 465.

[b] Ibid. edit. 3. p 501, 502.

[c] Ibid pag. 519.

[d] Ibid. pag. 52[illegible].

if thoſe two appearances were really of the ſame comet, the tranſverſe axis of the orbit of the comet would be near 18 times the axis of the earth's orbit; and the comet, when at its greateſt diſtance from the ſun, will be removed not leſs than 35 times as far as the middle diſtance of the earth.

8. AND this ſeems to be the ſhorteſt period of any of the comets. But it will be farther confirmed, if the ſame comet ſhould return a third time after another period of 75 years. However it is not to be expected, that comets ſhould preſerve the ſame regularity in their periods, as the planets; becauſe the great eccentricity of their orbits makes them liable to ſuffer very conſiderable alterations from the action of the planets, and other comets, upon them.

9. IT is therefore to prevent too great diſturbances in their motions from theſe cauſes, as our author obſerves, that while the planets revolve all of them nearly in the ſame plane, the comets are diſpoſed in very different ones, and diſtributed over all parts of the heavens; that, when in their greateſt diſtance from the ſun, and moving ſloweſt, they might be removed as far as poſſible out of the reach of each other's action [a]. The ſame end is likewiſe farther anſwered in thoſe comets, which by moving ſloweſt in the aphelion, or remoteſt diſtance from the ſun, deſcend neareſt to it, by placing the aphelion of theſe at the greateſt height from the ſun [b].

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[a] Newt. Princ. philoſ. p. 525.

[b] Ibid.

10. OUR philoſopher being led by his principles, to explain the motions of the comets, in the manner now related, takes occaſion from thence to give us his thoughts upon their nature and uſe. For which end he proves in the firſt place, that they muſt neceſſarily be ſolid and compact bodies, and by no means any ſort of vapour or light ſubſtance exhaled from the planets or ſtars: becauſe at the near diſtance, to which ſome comets approach the ſun, it could not be, but the immenſe heat, to which they are expoſed, ſhould inſtantaneouſly diſperſe and ſcatter any ſuch light volatile ſubſtance [a]. In particular the forementioned comet of 1680 deſcended ſo near the ſun, as to come within a ſixth part of the ſun's diameter from the ſurface of it. In which ſituation it muſt have been expoſed, as appears by computation, to a degree of heat exceeding the heat of the ſun upon our earth no leſs than 28000 times; and therefore might have contracted a degree of heat 2000 times greater, than that of red hot iron [b]. Now a ſubſtance, which could endure ſo intenſe a heat, without being diſperſed in vapor, muſt needs be firm and ſolid.

11. IT is ſhewn likewiſe, that the comets are opake ſubſtances, ſhining by a reflected light, borrowed from the ſun [c]. This is proved from the obſervation, that comets, though they are approaching the earth, yet diminiſh in luſtre, if at the ſame time they recede from

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[a] Ibid. pag. 508.

[b] Ibid.

[c] Ibid. pag. 484.

the ſun; and on the contrary, are found to encreaſe daily in brightneſs, when they advance towards the ſun, though at the ſame time they move from the earth [a].

12. THE comets therefore in theſe reſpects reſemble the planets; that both are durable opake bodies, and both revolve about the ſun in conic ſections. But farther the comets, like our earth, are ſurrounded by an atmoſphere. The air we breath is called the earth's atmoſphere; and it is moſt probable, that all the other planets are inveſted with the like fluid. Indeed here a difference is found between the planets and comets. The atmoſpheres of the planets are of ſo fine and ſubtile a ſubſtance, as hardly to be diſcerned at any diſtance, by reaſon of the ſmall quantity of light which they reflect, except only in the planet Mars. In him there is ſome little appearance of ſuch a ſubſtance ſurrounding him, as ſtars which have been covered by him are ſaid to look ſomewhat dim a ſmall ſpace before his body comes under them, as if their light, when he is near, were obſtructed by his atmoſphere. But the atmoſpheres which ſurround the comets are ſo groſs and thick, as to reflect light very copiouſly. They are alſo much greater in proportion to the body they ſurround, than thoſe of the planets, if we may judge of the reſt from our air; for it has been obſerved of comets, that the bright light appearing in the middle of them, which

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[a] Ibid. pag. 482, 483.

is reflected from the ſolid body, is ſcarce a ninth or tenth part of the whole comet.

13. I ſpeak only of the heads of the comets, the moſt lucid part of which is ſurrounded by a fainter light, the moſt lucid part being uſually not above a ninth or tenth part of the whole in breadth [a]. Their tails are an appearance very peculiar, nothing of the ſame nature appertaining in the leaſt degree to any other of the celeſtial bodies. Of that appearance there are ſeveral opinions; our author reduces them to three [b]. The two firſt, which he propoſes, are rejected by him; but the third he approves. The firſt is, that they ariſe from a beam of light tranſmitted through the head of the comet, in like manner as a ſtream of light is diſcerned, when the ſun ſhines into a darkened room through a ſmall hole. This opinion, as Sir Isaac Newton obſerves, implies the authors of it wholly unſkilled in the principles of optics; for that ſtream of light, ſeen in a darken'd room, ariſes from the reflection of the ſun beams by the duſt and motes floating in the air: for the rays of light themſelves are not ſeen, but by their being reflected to the eye from ſome ſubſtance, upon which they fall [c]. The next opinion examined by our author is that of the celebrated Des Cartes, who imagins theſe tails to be the light of the comet refracted in its paſſage to us, and thence affording an oblong repreſentation; as the light of the ſun does, when refracted

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[a] Ibid. pag. 481.

[b] Ibid. pag. 509.

[c] See the fore-cited place.

by the priſm in that noted experiment, which will have a great ſhare in the third book of this diſcourſe[a]. But this opinion is at once overturned from this conſideration only, that the planets could be no more free from this refraction than the comets; nay ought to have larger or brighter tails, than they, becauſe the light of the planets is ſtrongeſt. However our author has thought proper to add ſome farther objections againſt this opinion: for inſtance, that theſe tails are not variegated with colours, as is the image produced by the priſm, and which is inſeparable from that unequal refraction, which produces that diſproportioned length of the image. And beſides, when the light in its paſſage from different comets to the earth deſcribes the ſame path through the heavens, the refraction of it ſhould of neceſſity be in all reſpects the ſame. But this is contrary to obſervation; for the comet in 1680, the 28th day of December, and a former comet in the year 1577, the 29th day of December, appear'd in the ſame place of the heavens, that is, were ſeen adjacent to the ſame fixed ſtars, the earth likewiſe being in the ſame place at both times; yet the tail of the latter comet deviated from the oppoſition to the ſun a little to the northward, and the tail of the former comet declined from the oppoſition of the ſun five times as much ſouthward[b].

14. THERE are ſome other falſe opinions, though leſs regarded than theſe, which have been advanced upon this

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[a] Ibid. and Carteſ. Princ. Phil. part. 3. § 134, &c.

[b] Vid. Phil. Nat. princ. Math. p. 511.

argument. Theſe our excellent author paſſes over, haſtening to explain, what he takes to be the true cauſe of this appearance. He thinks it is certainly owing to ſteams and vapours exhaled from the body, and groſs atmoſphere of the comets, by the heat of the ſun; becauſe all the appearances agree perfectly to this ſentiment. The tails are but ſmall, while the comet is deſcending to the ſun, but enlarge themſelves to an immenſe degree, as ſoon as ever the comet has paſſed its perihelion; which ſhews the tail to depend upon the degree of heat, which the comet receives from the ſun. And that the intenſe heat to which comets, when neareſt the ſun, are expoſed, ſhould exhale from them a very copious vapour, is a moſt reaſonable ſuppoſition; eſpecially if we conſider, that in thoſe free and empty regions ſteams will more eaſily aſcend, than here upon the ſurface of the earth, where they are ſuppreſſed and hindered from riſing by the weight of the incumbent air: as we find by experiments made in veſſels exhauſted of the air, where upon removal of the air ſeveral ſubſtances will fume and diſcharge ſteams plentifully, which emit none in the open air. The tails of comets, like ſuch a vapour, are always in the plane of the comet's orbit, and oppoſite to the ſun, except that the upper part thereof inclines towards the parts, which the comet has left by its motion; reſembling perfectly the ſmoak of a burning coal, which, if the coal remain fixed, aſcends from it perpendicularly; but, if the coal be in motion, aſcends obliquely, inclining from the motion of the coal. And beſides, the tails of comets may be compared to this ſmoak in another reſpect, that both of them are denſer and more compact on the convex ſide, than on the concave. The different appearance of the head of the comet, after it has paſt its perihelion, from what it had before, confirms greatly this opinion of their tails: for ſmoke raiſed by a ſtrong heat is blacker and groſſer, than when raiſed by a leſs; and accordingly the heads of comets, at the ſame diſtance from the ſun, are obſerved leſs bright and ſhining after the perihelion, than before, as if obſcured by ſuch a groſs ſmoke.

15. THE obſervations of HEVELIUS upon the atmoſpheres of comets ſtill farther illuſtrate the ſame; who relates, that the atmoſpheres, eſpecially that part of them next the ſun, are remarkably contracted when near the ſun, and dilated again afterwards.

16. To give a more full idea of theſe tails, a rule is laid down by our author, whereby to determine at any time, when the vapour in the extremity of the tail firſt roſe from the head of the comet. By this rule it is found, that the tail does not conſiſt of a fleeting vapour, diſſipated ſoon after it is raiſed, but is of long continuance; that almoſt all the vapour, which roſe about the time of the perihelion from the comet of 1680, continued to accompany it, aſcending by degrees, being ſucceeded conſtantly by freſh matter, which rendered the tail contiguous to the comet. From this computation the tails are found to participate of another property of aſcending vapours, that when they aſcend with the greateſt velocity, they are leaſt incurvated.

17. The only objection that can be made againſt this opinion is the difficulty of explaining, how a ſufficient quantity of vapour can be raiſed from the atmoſphere of a comet to fill thoſe vaſt ſpaces, through which their tails are ſometimes extended. This our author removes by the following computation: our air being an elaſtic fluid, as has been ſaid before [a], is more denſe here near the ſurface of the earth, where it is preſſed upon by the whole air above; than it is at a diſtance from the earth, where it has a leſs weight incumbent. I have obſerved, that the denſity of the air is reciprocally proportional to the compreſſing weight. From hence our author computes to what degree of rarity the air muſt be expanded, according to this rule, at an height equal to a ſemidiameter of the earth: and he finds, that a globe of ſuch air, as we breath here on the ſurface of the earth, which ſhall be one inch only in diameter, if it were expanded to the degree of rarity, which the air muſt have at the height now mentioned, would fill all the planetary regions even to the very ſphere of Saturn, and far beyond. Now ſince the air at a greater height will be ſtill immenſly more rarified, and the ſurface of the atmoſpheres of comets is uſually about ten times the diſtance from the center of the comet, as the ſurface of the comet it ſelf, and the tails are yet vaſtly farther removed from the center of the comet; the vapour, which compoſes thoſe tails, may very well be allowed to be ſo expanded, as that a moderate quantity of matter may fill all that ſpace, they are ſeen to take up. Though indeed the atmoſpheres of comets being

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[a] Book I. Ch. 4. § 11.

very groſs, they will hardly be rarified in their tails to ſo great a degree, as our air under the ſame circumſtances; eſpecially ſince they may be ſomething condenſed, as well by their gravitation to the ſun, as that the parts will gravitate to one another; which will hereafter be ſhewn to be the univerſal property of all matter [a]. The only ſcruple left is, how ſo much light can be reflected from a vapour ſo rare, as this computation implies. For the removal of which our author obſerves, that the moſt refulgent of theſe tails hardly appear brighter, than a beam of the ſun's light tranſmitted into a darkened room through a hole of a ſingle inch diameter; and that the ſmalleſt fixed ſtars are viſible through them without any ſenſible diminution of their luſtre.

18. ALL theſe conſiderations put it beyond doubt, what is the true nature of the tails of comets. There has indeed nothing been ſaid, which will account for the irregular figures, in which thoſe tails are ſometimes reported to have appeared; but ſince none of thoſe appearances have ever been recorded by aſtronomers, who on the contrary aſcribe the ſame likeneſs to the tails of all comets, our author with great judgment refers all thoſe to accidental refractions by intervening clouds, or to parts of the milky way contiguous to the comets [b].

19. THE diſcuſſion of this appearance in comets has led Sir ISAAC NEWTON into ſome ſpeculations relating to their uſe, which I cannot but extreamly admire, as

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[a] Ch. 5.

[b] All theſe arguments are laid down in Philoſ. Nat. Princ. Lib. III. from p. 509. to 517.

repreſenting in the ſtrongeſt light imaginable the extenſive providence of the great author of nature, who, beſides the furniſhing this globe of earth, and without doubt the reſt of the planets, ſo abundantly with every thing neceſſary for the ſupport and continuance of the numerous races of plants and animals, they are ſtocked with, has over and above provided a numerous train of comets, far exceeding the number of the planets, to rectify continually, and reſtore their gradual decay, which is our author's opinion concerning them [a]. For ſince the comets are ſubject to ſuch unequal degrees of heat, being ſometimes burnt with the moſt intenſe degree of it, at other times ſcarce receiving any ſenſible influence from the ſun; it can hardly be ſuppoſed, they are deſigned for any ſuch conſtant uſe, as the planets. Now the tails, which they emit, like all other kinds of vapour, dilate themſelves as they aſcend, and by conſequence are gradually diſperſed and ſcattered through all the planetary regions, and thence cannot but be gathered up by the planets, as they paſs through their orbs: for the planets having a power to cauſe all bodies to gravitate towards them, as will in the ſequel of this diſcourſe be ſhewn [b]; theſe vapours will be drawn in proceſs of time into this or the other planet, which happens to act ſtrongeſt upon them. And by entering the atmoſpheres of the earth and other planets, they may well be ſuppoſed to contribute to the renovation of the face of things, in particular to ſupply the diminution cauſed in the humid parts by vegetation and putrefaction. For vegetables are nouriſhed by moiſture, and by putrefaction are turned in great part into dry earth; and an earthy ſubſtance always ſubſides in fermenting liquors; by which means the dry parts of the planets muſt continually increaſe, and the fluids diminiſh, nay in a ſufficient length of time be exhauſted, if not ſupplied by ſome ſuch means. It is farther our great author's opinion, that the moſt ſubtile and active parts of the air, upon which the life of things chiefly depends, is derived to us, and ſupplied by the comets. So far are they from portending any hurt or miſchief to us, which the natural fears of men are ſo apt to ſuggeſt from the appearance of any thing uncommon and aſtoniſhing.

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[a] Philoſ. Nat. Princ. Lib. III. p. 515.

[b] Ch. 5.

20. THAT the tails of comets have ſome ſuch important uſe ſeems reaſonable, if we conſider, that thoſe bodies do not ſend out thoſe fumes merely by their near approach to the ſun; but are framed of a texture, which diſpoſes them in a particular manner to fume in that ſort: for the earth, without emitting any ſuch ſteam, is more than half the year at a leſs diſtance from the ſun, than the comet of 1664 and 1665 approached it, when neareſt; likewiſe the comets of 1682 and 1683 never approached the ſun much above a ſeventh part nearer than Venus, and were more than half as far again from the ſun as Mercury; yet all theſe emitted tails.

21. FROM the very near approach of the comet of 1680 our author draws another ſpeculation; for if the

ſun have an atmoſphere about it, the comet mentioned ſeems to have deſcended near enough to the ſun to enter within it. If ſo, it muſt have been ſomething retarded by the reſiſtance it would meet with, and conſequently in its next deſcent to the ſun will fall nearer than now; by which means it will meet with a greater reſiſtance, and be again more retarded. The event of which muſt be, that at length it will impinge upon the ſun's ſurface, and thereby ſupply any decreaſe, which may have happened by ſo loſing an emiſſion of light, or otherwiſe. And ſomething like this our author conjectures may be the caſe of thoſe fixed ſtars, which by an additional increaſe of their luſtre have for a certain time become viſible to us, though uſually they are out of ſight. There is indeed a kind of fixed ſtars, which appear and diſappear at regular and equal intervals: here ſome more ſteady cauſe muſt be ſought for; perhaps theſe ſtars turn round their own axis's, as our ſun does; and have ſome part of their body more luminous than the other, whereby they are ſeen, when the moſt lucid part is next to us, and when the darker part is turned toward us, they vaniſh out of ſight.

22. WHETHER the ſun does really diminiſh, as has been here ſuggeſted, is difficult to prove; yet that it either does ſo, or that the earth increaſes, if not both, is rendered probable from Dr. HALLEY's obſervation [b], that by comparing

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[a] See Ch. 1. § 11.

[b] Newt. Princ. Philoſ. pag. 525, 526. An account of all the ſtars of both theſe kinds, which have appeared within the laſt 150 years may be ſeen in the Philoſophical tranſactions, vol. 29. numb. 346.

the proportion, which the periodical time of the moon bore to that of the ſun in former times, with the proportion between them at preſent, the moon is found to be ſomething accelerated in reſpect of the ſun. But if the ſun diminiſh, the periods of the primary planets will be lengthened; and if the earth be encreaſed, the period of the moon will be ſhortened: as will appear by the next chapter, wherein it ſhall be ſhewn, that the power of the ſun and earth is the reſult of the ſame power being lodg'd in all their parts, and that this principle of producing gravitation in other bodies is proportional to the ſolid matter in each body.

CHAP. V.

Of the BODIES of the SUN and PLANETS.

OUR author, after having diſcovered that the celeſtial motions are performed by a force extended from the ſun and primary planets, follows this power into the deepeſt receſſes of thoſe bodies themſelves, and proves the ſame to accompany the ſmalleſt particle, of which they are compoſed.

2. PREPARATIVE hereto he ſhews firſt, that each of the heavenly bodies attracts the reſt, and all bodies, with ſuch different degrees of force, as that the force of the ſame at-

tracting body is exerted on others exactly in proportion to the quantity of matter in the body attracted [a].

3. OF this the firſt proof he brings is from experiments made here upon the earth. The power by which the moon is influenced was above ſhewn to be the ſame, with that power here on the ſurface of the earth, which we call gravity [b]. Now one of the effects of the principle of gravity is, that all bodies deſcend by this force from the ſame height in equal times. Which has been long taken notice of; particular methods having been invented to ſhew that the only cauſe, why ſome bodies were obſerved to fall from the ſame height ſooner than others, was the reſiſtance of the air. This we have above related [c]; and proved from hence, that ſince bodies reſiſt to any change of their ſtate from reſt to motion, or from motion to reſt, in proportion to the quantity of matter contained in them; the power that can move different quantities of matter equally, muſt be proportional to the quantity. The only objection here is, that it can hardly be made certain, whether this proportion in the effect of gravity on different bodies holds perfectly exact or not from theſe experiments; by reaſon that the great ſwiftneſs, with which bodies fall, prevents our being able to determine the times of their deſcent with all the exactneſs requiſite. Therefore to remedy this inconvenience, our author ſubſtitutes another more certain experiment in the room of theſe made upon falling bodies. Pen-

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[a] Newt. Princ. Philoſ. Nat. Lib. III. prop. 6.

[b] Ch. 3. § 6.

[c] Book I. Ch. 2. § 24.

dulums are cauſed to vibrate by the ſame principle, as makes bodies deſcend; the power of gravity putting them in motion, as well as the other. But if the ball of any pendulum, of the ſame length with another, were more or leſs attracted in proportion to the quantity of ſolid matter in the ball, that pendulum muſt accordingly move faſter or ſlower than the other. Now the vibrations of pendulums continue for a great length of time, and the number of vibrations they make may eaſily be determined without ſuſpicion of error; ſo that this experiment may be extended to what exactneſs one pleaſes: and our author aſſures us, that he examined in this way ſeveral ſubſtances, as gold, ſilver, lead, glaſs, ſand, common ſalt, wood, water, and wheat; in all which he found not the leaſt deviation from the proportion mentioned, though he made the experiment in ſuch a manner, that in bodies of the ſame weight a difference in the quantity of their matter leſs than a thouſandth part of the whole would have diſcovered it ſelf [a]. It appears therefore, that all bodies are made to deſcend by the power of gravity here, near the ſurface of the earth, with the ſame degree of ſwiftneſs. We have above obſerved this deſcent to be after the rate of 16 $\frac{1}{8}$ feet in the firſt ſecond of time from the beginning of their fall. Moreover it was alſo obſerved, that if any body, which fell here at the ſurface of the earth after this rate, were to be conveyed up to the height of the moon, it would

[a] Newt. Princ. Lib. III. prop. 6.

deſcend from thence juſt with the ſame degree of velocity, as that with which the moon is attracted toward the earth; and therefore the power of the earth upon the moon bears the ſame proportion to the power it would have upon thoſe bodies at the ſame diſtance, as the quantity of matter in the moon bears to the quantity in thoſe bodies.

4. Thus the aſſertion laid down is proved in the earth, that the power of the earth on every body it attracts is, at the ſame diſtance from the earth, proportional to the quantity of ſolid matter in the body acted on. As to the ſun, it has been ſhewn, that the power of the ſun's action upon the ſame primary planet is reciprocally in the duplicate proportion of the diſtance; and that the power of the ſun decreaſes throughout in the ſame proportion, the motion of comets traverſing the whole planetary region teſtifies. This proves, that if any planet were removed from the ſun to any other diſtance whatever, the degree of its acceleration toward the ſun would yet remain reciprocally in the duplicate proportion of its diſtance. But it has likewiſe been ſhewn, that the degree of acceleration, which the ſun gives to every one of the planets, is reciprocally in the duplicate proportion of their reſpective diſtances. All which compared together puts it out of doubt, that the power of the ſun upon any planet, removed into the place of any other, would give it the ſame velocity of deſcent, as it gives that other; and conſequently, that the ſun's action upon different planets at the ſame diſtance would be proportional to the quantity of matter in each. It has farther

been ſhewn, that the ſun attracts the primary planets, and their reſpective ſecondary, when at the ſame diſtance, ſo as to communicate to both the ſame degree of velocity; and therefore the force, wherewith the ſun acts on the ſecondary planet, bears the ſame proportion to the force, wherewith at the ſame diſtance it attracts the primary, as the quantity of ſolid matter in the ſecondary planet bears to the quantity of matter in the primary.

5. THIS property therefore is proved of both kinds of planets, in reſpect of the ſun. Therefore the ſun poſſeſſes the quality found in the earth, of acting on bodies with a degree of force proportional to the quantity of matter in the body, which receives the influence.

6. THAT the power of attraction, with which the other planets are endued, ſhould differ from that of the earth, can hardly be ſuppoſed, if we conſider the ſimilitude between thoſe bodies; and that it does not in this reſpect, is farther proved from the ſatellites of Saturn and Jupiter, which are attracted by their reſpective primary according to the ſame law, that is, in the ſame proportion to their diſtances, as the primary are attracted by the ſun: ſo that what has been concluded of the ſun in relation to the primary planets, may be juſtly concluded of theſe primary in reſpect of their ſecondary, and in conſequence of that, in regard likewiſe to all other bodies, viz. that they will attract every body in proportion to the quantity of ſolid matter it contains.

7. HENCE it follows, that this attraction extends itſelf to every particle of matter in the attracted body: and that no portion of matter whatever is exempted from the influence of thoſe bodies, to which we have proved this attractive power to belong.

8. BEFORE we proceed farther, we may here remark, that this attractive power both of the ſun and planets now appears to be quite of the ſame nature in all; for it acts in each in the ſame proportion to the diſtance, and in the ſame manner acts alike upon every particle of matter. This power therefore in the ſun and other planets is not of a different nature from this power in the earth; which has been already ſhewn to be the ſame with that, which we call gravity [a].

9. AND this lays open the way to prove, that the attracting power lodged in the ſun and planets, belongs likewiſe to every part of them: and that their reſpective powers upon the ſame body are proportional to the quantity of matter, of which they are compoſed; for inſtance, that the force with which the earth attracts the moon, is to the force, with which the ſun would attract it at the ſame diſtance, as the quantity of ſolid matter contained in the earth, to the quantity contained in the ſun [b].

10. THE firſt of theſe aſſertions is a very evident conſequence from the latter. And before we proceed to the proof,

[a] Ch. 3. § 6.

[b] Newt. Princ. philoſ. Lib. III. prop. 7. cor. 1.

it muſt firſt be ſhewn, that the third law of motion, which makes action and reaction equal, holds in theſe attractive powers. The moſt remarkable attractive force, next to the power of gravity, is that, by which the loadſtone attracts iron. Now if a loadſtone were laid upon water, and ſupported by ſome proper ſubſtance, as wood or cork, ſo that it might ſwim; and if a piece of iron were cauſed to ſwim upon the water in like manner: as ſoon as the loadſtone begins to attract the iron, the iron ſhall move toward the ſtone, and the ſtone ſhall alſo move toward the iron; when they meet, they ſhall ſtop each other, and remain fixed together without any motion. This ſhews, that the velocities, wherewith they meet, are reciprocally proportional to the quantities of ſolid matter in each; and that by the ſtone's attracting the iron, the ſtone itſelf receives as much motion, in the ſtrict philoſophic ſenſe of that word[a], as it communicates to the iron: for it has been declared above to be an effect of the percuſſion of two bodies, that if they meet with velocities reciprocally proportional to the reſpective bodies, they ſhall be ſtopped by the concourſe, unleſs their elaſticity put them into freſh motion; but if they meet with any other velocities, they ſhall retain ſome motion after meeting[b]. Amber, glaſs, ſealing-wax, and many other ſubſtances acquire by rubbing a power, which from its having been remarkable, particularly in amber, is called electrical. By this power they will for ſome time after rubbing attract light bodies, that ſhall be brought within the ſphere of their activity. On the other hand Mr. BOYLE found, that if a piece of amber be hung in a perpendicular poſition by a ſtring, it ſhall be drawn itſelf toward the body whereon it was rubbed, if that body be brought near it. Both in the loadſtone and in electrical bodies we uſually aſcribe the power to the particular body, whoſe preſence we find neceſſary for producing the effect. The loadſtone and any piece of iron will draw each other, but in two pieces of iron no ſuch effect is ordinarily obſerved; therefore we call this attractive power the power of the loadſtone: though near a loadſtone two pieces of iron will alſo draw each other. In like manner the rubbing of amber, glaſs, or any ſuch body, till it is grown warm, being neceſſary to cauſe any action between thoſe bodies and other ſubſtances, we aſcribe the electrical power to thoſe bodies. But in all theſe caſes if we would ſpeak more correctly, and not extend the ſenſe of our expreſſions beyond what we ſee; we can only ſay that the neighbourhood of a loadſtone and a piece of iron is attended with a power, whereby the loadſtone and the iron are drawn toward each other; and the rubbing of electrical bodies gives riſe to a power, whereby thoſe bodies and other ſubſtances are mutually attracted. Thus we muſt alſo underſtand in the power of gravity, that the two bodies are mutually made to approach by the action of that power. When the ſun draws any planet, that planet alſo draws the ſun; and the motion, which the planet receives from the ſun, bears the ſame proportion to the motion, which the ſun it ſelf receives, as the quantity of ſolid matter in the ſun bears to the quantity of ſolid matter in the planet. Hitherto, for brevity ſake in ſpeaking of theſe forces, we have generally aſcribed them to the body, which is leaſt moved; as when we called the power, which exerts itſelf between the ſun and any planet, the attractive power of the ſun; but to ſpeak more correctly, we ſhould rather call this power in any caſe the force, which acts between the ſun and earth, between the ſun and Jupiter, between the earth and moon, &c. for both the bodies are moved by the power acting between them, in the ſame manner, as when two bodies are tied together by a rope, if that rope ſhrink by being wet, or otherwiſe, and thereby cauſe the bodies to approach, by drawing both, it will communicate to both the ſame degree of motion, and cauſe them to approach with velocities reciprocally proportional to the reſpective bodies. From this mutual action between the ſun and planet it follows, as has been obſerved above [a], that the ſun and planet do each move about their common center of gravity. Let A (in fig. 108.) repreſent the ſun, B a planet, C their common center of gravity. If theſe bodies were once at reſt, by their mutual attraction they would directly approach each other with ſuch velocities, that their common center of gravity would remain at reſt, and the two bodies would at length meet in that point. If the planet B were to receive an impulſe, as in the direction of the line DE, this would prevent the two bodies from falling together;

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[a] See Book I. Ch. 1. § 25.

[b] Ibid. § 5, 6.

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[a] Chap. 2. § 8.

but their common center of gravity would be put into motion in the direction of the line CF equidiſtant from BE. In this caſe Sir ISAAC NEWTON proves[a], that the ſun and planet would deſcribe round their common center of gravity ſimilar orbits, while that center would proceed with an uniform motion in the line CF; and ſo the ſyſtem of the two bodies would move on with the center of gravity without end. In order to keep the ſyſtem in the ſame place, it is neceſſary, that when the planet received its impulſe in the direction BE, the ſun ſhould alſo receive ſuch an impulſe the contrary way, as might keep the center of gravity C without motion; for if theſe began once to move without giving any motion to their common center of gravity, that center would always remain fixed.

11. BY this may be underſtood in what manner the action between the ſun and planets is mutual. But farther, we have ſhewn above[b], that the power, which acts between the ſun and primary planets, is altogether of the ſame nature with that, which acts between the earth and the bodies at its ſurface, or between the earth and its parts, and with that which acts between the primary planets and their ſecondary; therefore all theſe actions muſt be aſcribed to the ſame cauſe[c]. Again, it has been already proved, that in different planets the force of the ſun's action upon each at the ſame diſtance would be proportional to the quantity of ſolid matter in the planet[d]; therefore the reaction of each planet

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[a] Newt. Princ. Lib. I. prop. 63. [b] § 8. [c] See Introd. § 23. [d] § 4, 5.

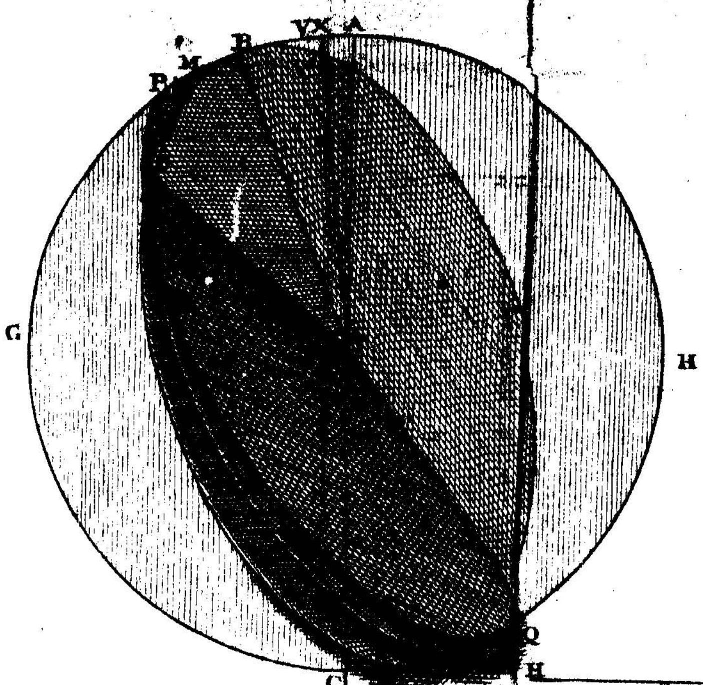


Fig. 103.

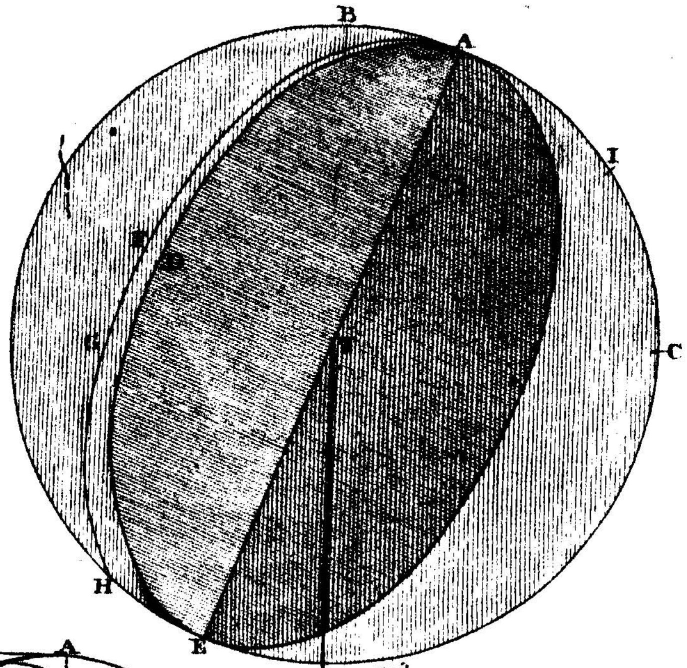


Fig. 104.

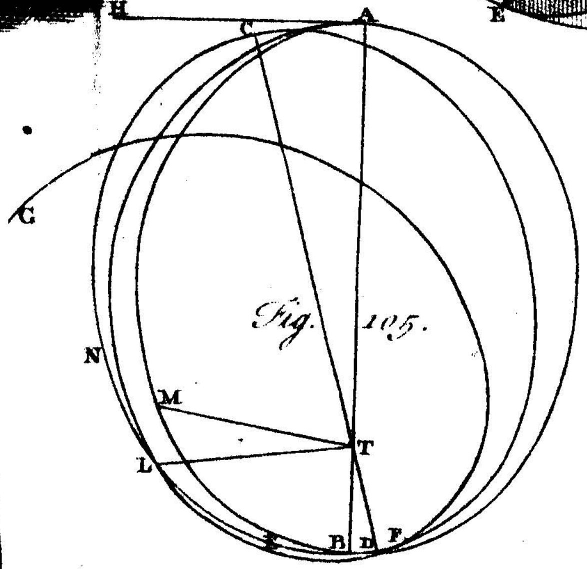


Fig. 105.

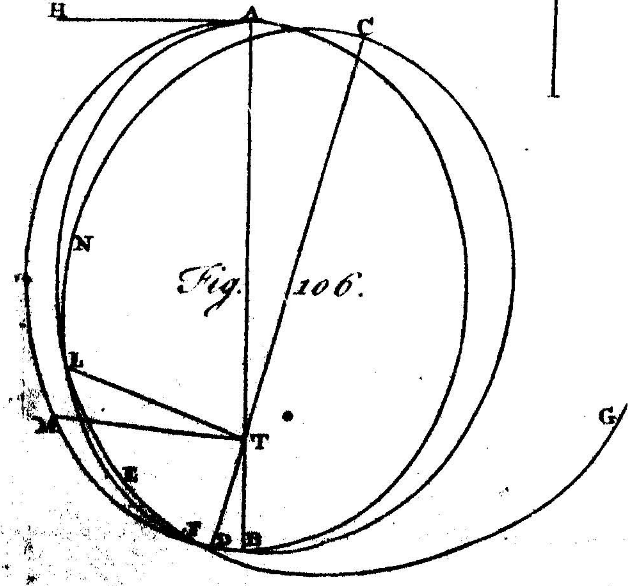
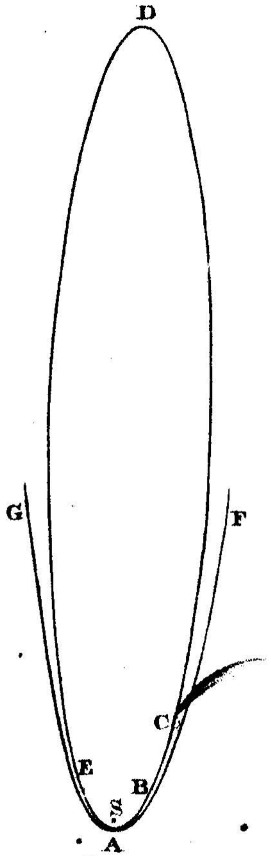


Fig. 106.



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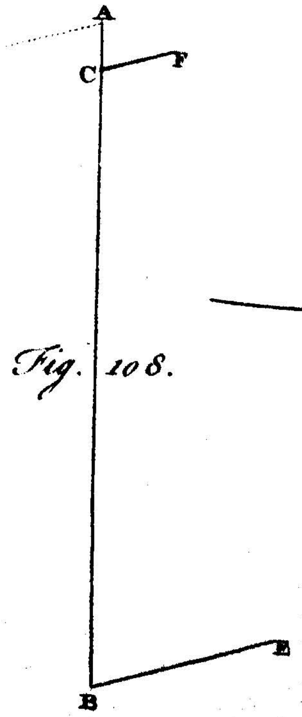

Fig. 108.

on the ſun at the ſame diſtance, or the motion, which the ſun would receive from each planet, would alſo be proportional to the quantity of matter in the planet; that is, theſe planets at the ſame diſtance would act on the ſame body with degrees of ſtrength proportional to the quantity of ſolid matter in each.

12. IN the next place, from what has been now proved, our great author has deduced this farther conſequence, no leſs ſurprizing than elegant; that each of the particles, out of which the bodies of the ſun and planets are framed, exert their power of gravitation by the ſame law, and in the ſame proportion to the diſtance, as the great bodies which they compoſe. For this purpoſe he firſt demonſtrates, that if a globe were compounded of particles, which will attract the particles of any other body reciprocally in the duplicate proportion of their diſtances, the whole globe will attract the ſame in the reciprocal duplicate proportion of their diſtances from the center of the globe; provided the globe be of uniform denſity throughout [a]. And from this our author deduces the reverſe, that if a globe acts upon diſtant bodies by the law juſt now ſpecified, and the power of the globe is derived from its being compoſed of attractive particles; each of thoſe particles will attract after the ſame proportion [b]. The manner of deducing this is not ſet down at large by our author, but is as follows. The globe is

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[a] Newt. Princ. philoſ. Lib. I. prop. 74

[b] Ibid. coroll. 3.

ſuppoſed to act upon the particles of a body without it conſtantly in the reciprocal duplicate proportion of their diſtances from its center; and therefore at the ſame diſtance from the globe, on which ſide ſoever the body be placed, the globe will act equally upon it. Now becauſe, if the particles, of which the globe is compoſed, acted upon thoſe without in the reciprocal duplicate proportion of their diſtances, the whole globe would act upon them in the ſame manner as it does; therefore, if the particles of the globe have not all of them that property, ſome muſt act ſtronger than in that proportion, while others act weaker: and if this be the condition of the globe, it is plain, that when the body attracted is in ſuch a ſituation in reſpect of the globe, that the greater number of the ſtrongeſt particles are neareſt to it, the body will be more forcibly attracted; than when by turning the globe about, the greater quantity of weak particles ſhould be neareſt, though the diſtance of the body ſhould remain the ſame from the center of the globe. Which is contrary to what was at firſt remarked, that the globe on all ſides of it acts with the ſame ſtrength at the ſame diſtance. Whence it appears, that no other conſtitution of the globe can agree to it.

13. FROM theſe propoſitions it is farther collected, that if all the particles of one globe attract all the particles of another in the proportion ſo often mentioned, the attracting globe will act upon the other in the ſame proportion to the diſtance between the center of the globe which attracts, and the center of that which is attracted[a]: and farther, that this

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[a] Lib. I. Prop. 75. and Lib. III. prop. 8.

proportion holds true, though either or both the globes be compoſed of diſſimilar parts, ſome rarer and ſome more denſe; provided only, that all the parts in the ſame globe equally diſtant from the center be homogeneous [a]. And alſo, if both the globes attract each other [b]. All which place it beyond contradiction, that this proportion obtains with as much exactneſs near and contiguous to the ſurface of attracting globes, as at greater diſtances from them.

14. THUS our author, without the pompous pretence of explaining the cauſe of gravity, has made one very important ſtep toward it, by ſhewing that this power in the great bodies of the univerſe, is derived from the ſame power being lodged in every particle of the matter which compoſes them: and conſequently, that this property is no leſs than univerſal to all matter whatever, though the power be too minute to produce any viſible effects on the ſmall bodies, wherewith we converſe, by their action on each other [c]. In the fixed ſtars indeed we have no particular proof that they have this power; for we find no apperance to demonſtrate that they either act, or are acted upon by it. But ſince this power is found to belong to all bodies, whereon we can make obſervation; and we ſee that it is not to be altered by any change in the form of bodies, but always accompanies them in every ſhape without diminution, remaining ever proportional to the quantity of ſolid matter in each; ſuch a power muſt without doubt belong univerſally to all matter.

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[a] Lib. I. Prop. 76. [b] Ibid. cor. 5. [c] Vid Lib. III. Prop. 7. coroll. 1.

15. THIS

15. This therefore is the univerſal law of matter; which recommends it ſelf no leſs for its great plainneſs and ſimplicity, than for the ſurprizing diſcoveries it leads us to. By this principle we learn the different weight, which the ſame body will have upon the ſurfaces of the ſun, and of diverſe planets; and by the ſame we can judge of the compoſition of thoſe celeſtial bodies, and know the denſity of each; which is formed of the moſt compact, and which of the moſt rare ſubſtance. Let the adverſaries of this philoſophy reflect here, whether loading this principle with the appellation of an occult quality, or perpetual miracle, or any other reproachful name, be ſufficient to diſſuade us from cultivating it; ſince this quality, which they call occult, leads to the knowledge of ſuch things, that it would have been reputed no leſs than madneſs for any one, before they had been diſcovered, even to have conjectured that our faculties ſhould ever have reached ſo far.

16. See how all this naturally follows from the foregoing principles in thoſe planets, which have ſatellites moving about them. By the times, in which theſe ſatellites perform their revolutions, compared with their diſtances from their reſpective primary, the proportion between the power, with which one primary attracts his ſatellites, and the force with which any other attracts his will be known; and the proportion of the power with which any planet attracts its ſecondary, to the power with which it attracts a body at its ſurface is found, by comparing the diſtance of the ſecondary planet from the center of the primary, to the